CHAPTER 5

Systems of Equations and Inequalities

Continue to think about your interactions with your student regarding mathematics. Remember to encourage your student to become an independent learner and thinker by asking him or her to do the explaining. If you are explaining, you are not giving your student a chance to see what they don't understand.

Content Summary

Chapter 5 reinforces ideas of linearity from Chapter 3. Chapter 5 presents problems modeled by more than one linear equation at a time and then considers problems represented by linear inequalities.

Systems of Linear Equations

Many real-world problems involve situations in which two or more values change linearly at the same time. Often you want to find when those values will be equal. For example, the values might be the location of two walkers, and you want to determine when the walkers meet.

Each growing value is represented by a linear equation. So several values changing linearly are represented by several linear equations, called a *system* of linear equations. Identifying where the values are equal requires solving the system, that is, finding values of the variables that make all the linear equations in the system true.

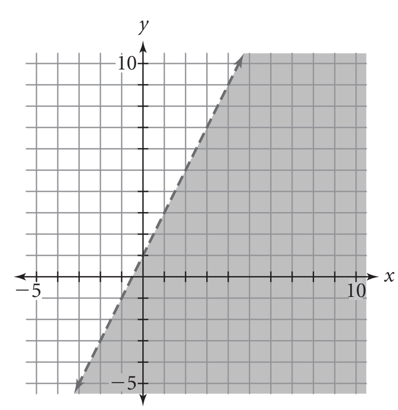
Discovering Algebra includes four methods for solving systems of two linear equations: by graphing, by substitution, by elimination, and by matrices.

Linear Inequalities

Another way to build conceptually on the idea of a linear equation is to change the equal sign to a less than or greater than sign. If you do that, you are stating that the two expressions are *not* equivalent.

This chapter introduces linear inequalities in which one of the variables has a known value, such as $5 + 2a > 21$. Such an inequality has infinitely many solutions, and you can visualize them on a number line. Students find the solutions by using the same methods they used for solving a linear equation in Chapter 3.

Then this chapter considers a linear inequality in two variables. Just as the pairs of numbers satisfying an equation can be represented by a line on a graph, the pairs of numbers satisfying an inequality can be represented on a graph. They appear as all points on one side of the line representing the corresponding equation. For a strict inequality, such as $y < 2x + 1$, points on the boundary line $y = 2x + 1$ make the inequality false, so the line is dashed to show that only the shaded portion of the graph, and not the boundary line, represents the solution.



Systems of Linear Inequalities

The mathematical methods called *linear programming* apply to many real-world situations. These methods rely on systems of linear inequalities. You can visualize the solutions to these systems graphically as the region containing only points that satisfy all inequalities in the system.

(continued)

Summary Problem

This summary problem is based on Lesson 5.4, Exercise 10. Will is baking bread. He has two different kinds of flour. Flour X is enriched with 0.12 mg of calcium per gram; Flour Y is enriched with 0.04 mg of calcium per gram. Each loaf has 300 g of flour, and Will wants each loaf to have 30 mg of calcium. How much of each type of flour should he use for each loaf?

If your student's teacher covered Lesson 5.7, add the following modification: Will has started selling his bread and is having trouble making a profit. To keep his overall costs down, each loaf can contain *at most* 300 g of flour. He would like at least 25 g of calcium per loaf.

Discuss these questions and scenarios with your student in your role as student to your student:

- Write a system of equations to represent the problem and explain the meaning of each variable and each equation in the context of the problem.
- Which method for solving systems of equations would you choose to solve the system?
- Solve the system using one method. Then check by solving the system using a second method.
- Suppose Will is almost out of flour. He only has 275 g remaining. Can he still bake a loaf of bread with 30 mg of calcium?

Sample Answers

One possible system of equations is:

$$\begin{cases} x + y = 300 \\ 0.12x + 0.04y = 30 \end{cases}$$

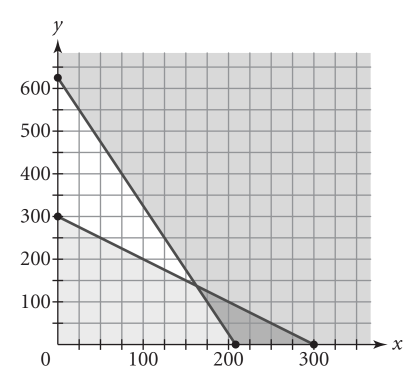
The variable x represents the amount of Flour X in grams, and y represents the amount of Flour Y in grams. The first equation represents the restriction that each loaf has 300 g of flour. $0.12x$ represents the amount of calcium contributed by Flour X, and $0.04y$ represents the amount of calcium contributed by Flour Y. The second equation represents the total amount of calcium in the loaf.

Substitution, elimination, and matrices are probably the best choices for solving this system. Students might also choose to solve by graphing on their calculator. You might ask your student to solve this by different methods as you are reviewing different lessons. They should find that the loaf of bread requires 225 g of Flour X and 75 g of Flour Y. If the problem is changed to use only 275 g of flour, the solution will be 237.5 g of Flour X and 37.5 g of Flour Y.

If Will can use no more than 300 g of flour, and each loaf must have at least 25 g of calcium, the system of inequalities can represent the new situation:

$$\begin{cases} x + y \leq 300 \\ 0.12x + 0.04y \geq 25 \end{cases}$$

Students should test points to determine how to shade each inequality, and then find several points that satisfy both inequalities. Some sample solutions are (180, 100), representing 180 g of Flour X and 100 g of Flour Y, (215, 16), and (290, 0).



Chapter 5 • Review Exercises

Name ______________________ **Period** __________ **Date** ______________

1. *(Lesson 5.1)* Determine whether the ordered pair is a solution to the system of equations. Graph both lines in the system, and plot the point.

a. $(1, -2)$ $\begin{cases} y = 2x - 4 \\ y = -x - 1 \end{cases}$

b. $(3, 1)$ $\begin{cases} y = -\frac{2}{3}x + 3 \\ y = \frac{2}{3}x - 2 \end{cases}$

2. *(Lesson 5.2)* Solve this system of equations using the substitution method, and then check your answer.

$\begin{cases} 2x + 3y = 7 \\ x - 4y = -2 \end{cases}$

3. *(Lesson 5.3)* Solve this system by elimination, and then check your work.

$\begin{cases} 2x - 3y = -2 \\ 5x + 2y = -5 \end{cases}$

4. *(Lessons 5.1–5.3)* Jenna purchased peaches and pears at the local market. The peaches cost $2.90 per pound, and the pears cost $1.10 per pound. Jenna bought a total of 8 pounds of fruit, which cost $18.34. How many pounds of each fruit did Jenna buy?

5. *(Lesson 5.5)* Solve the inequality $3 - 5x \geq 8$ and graph the solutions on a number line.

6. *(Lessons 5.6, 5.7)* Consider the system of inequalities $\begin{cases} y < 2x - 3 \\ y \geq -x + 1 \end{cases}$.

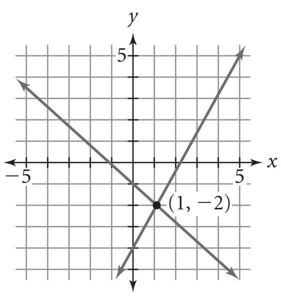
a. Determine whether each of the following ordered pairs is a solution to this system of inequalities.

i. $(0, 2)$ **ii.** $(4, 2)$ **iii.** $(-3, 1)$ **iv.** $(3, -2)$

b. Graph the system of inequalities, and plot each of the points from 6a.

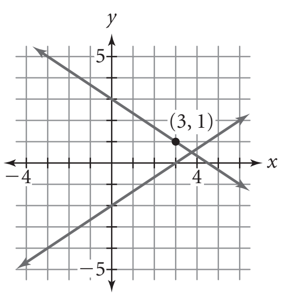
1. a. Yes. The ordered pair $(1, -2)$ satisfies both equations.

$$y = 2x - 4 \qquad y = -x - 1$$
$$-2 \stackrel{?}{=} 2(1) - 4 \qquad -2 \stackrel{?}{=} -1 - 1$$
$$-2 = -2 \qquad -2 = -2$$



b. No. The ordered pair $(3, 1)$ does not satisfy the second equation.

$$y = -\frac{2}{3}x + 3 \qquad y = \frac{2}{3}x - 2$$
$$1 \stackrel{?}{=} -\frac{2}{3}(3) + 3 \qquad 1 \stackrel{?}{=} \frac{2}{3}(3) - 2$$
$$1 \stackrel{?}{=} -2 + 3 \qquad 1 \stackrel{?}{=} 2 - 2$$
$$1 = 1 \qquad 1 \neq 0$$



2. $(2, 1)$. Solve the second equation for x, and substitute the expression into the first equation.

$x - 4y = -2$	Second equation.
$x = 4y - 2$	Add $4y$ to both sides.
$2(4y - 2) + 3y = 7$	Substitute $4y - 2$ for x in the first equation.
$8y - 4 + 3y = 7$	Use the distributive property.
$11y - 4 = 7$	Combine like terms.
$11y = 11$	Add 4 to both sides.
$y = 1$	Divide both sides by 11.

Substitute 1 for y into one of the original equations and solve for x: $x - 4(1) = -2$; $x = 2$. The solution is $(2, 1)$.

Check your solution.

$$2x + 3y = 7 \qquad x - 4y = -2$$
$$2(2) + 3(1) \stackrel{?}{=} 7 \qquad 2 - 4(1) \stackrel{?}{=} -2$$
$$7 = 7 \qquad -2 = -2$$

3. $(-1, 0)$

$2(2x - 3y) = 2(-2) \rightarrow 4x - 6y = -4$	Multiply both sides by 2.
$3(5x + 2y) = 3(-5) \rightarrow \underline{15x + 6y = -15}$	Multiply both sides by 3.
$19x = -19$	Add the equations.
$x = -1$	Divide both sides by 19.

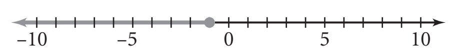
Substitute this x-value into either of the original equations and solve to find y. Using the first equation, $2(-1) - 3y = -2$; $y = 0$.

Check your solution in both equations.

4. 5.3 lb of peaches, 2.7 lb of pears. Let x be the number of pounds of peaches, and y be the number of pounds of pears Jenna bought. Adding up pounds yields the equation $x + y = 8$, and adding up cost yields the equation $2.90x + 1.10y = 18.34$. This system may be more easily solved by substitution.

5.

$3 - 5x \geq 8$	Original inequality.
$-5x \geq 5$	Subtract 3 from both sides.
$x \leq -1$	Divide both sides by -5 and reverse the inequality.



6. a. Substitute the values for x and y into each inequality and see whether a true statement results.

i. No. The ordered pair satisfies the second inequality but not the first.

ii. Yes. The ordered pair satisfies both inequalities.

iii. No. the ordered pair does not satisfy either inequality.

iv. Yes. The ordered pair satisfies both inequalities.

b. Begin by graphing the equations $y = 2x - 3$ and $y = 1 - x$. Because the first inequality has $<$ instead of $\leq$, the line $y = 2x - 3$ should be dashed to indicate that it is not included in the solution. The other line should be solid. Plot the points and shade the area containing the solutions.